A sticky problem : use R to estimate the probability that a triangle can be formed from the three pieces that result when a straight stick is broken at random in two places. Notice that if the length of one of the three pieces is more than one-half of the length of the stick, then a triangle cannot be made, but otherwise it can.

A sticky solution : suppose the stick has unit length, that the two breaks are at $0 < u_1, u_2 < 1$, and let $d_1 = \min(u_1, u_2)$ and $d_2 = \max(u_1, u_2)$. A triangle can be made iff $\max(d_1, d_2 - d_1, 1 - d_2) < 0.5$, i..e, none of the pieces is longer than 0.5. We can estimate this by

```
n <- 10^7
u1 <- runif(n)
u2 <- runif(n)
d1 <- pmin(u1,u2)  # pmin and pmax give parallel minima and maxima
d2 <- pmax(u1,u2)
mean((pmax(d1,d2-d1,1-d2)<=0.5))
```

where the two breaks are independent and uniformly distributed along the stick. The probability sought seems to be close to 0.25.

An acute problem : use R to estimate the probability that the triangle formed from three random points in the plane is acute. As the radius of the circumcircle of the triangle is irrelevant and the problem is rotationally symmetric, we can take this circumcircle to be the unit circle in the complex plane, with one of the points at $e^{i0} = (1, 0)$.

An acute solution : The other two points can be placed at $e^{i\theta_1}$ and $e^{i\theta_2}$, where θ_1, θ_2 are uniform on $(0, 2\pi)$, and then the triangle is acute if and only if all the angles subtended at the origin are less than π. We can estimate the probability of this event by :

```
n <- 10^7
theta1 <- 2*pi*runif(n)
theta2 <- 2*pi*runif(n)
d1 <- pmin(theta1,theta2)  # pmin and pmax give parallel minima and maxima
d2 <- pmax(theta1,theta2)
mean((pmax(d1,d2-d1,2*pi-d2)<=pi))
```

The probability sought seems to be close to 0.25.. Curious, no ?